# Dark Matter Candidate Particles, *CP* Violation and Higgs Bosons

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**Abstract** In a previous paper, we proposed the infinite sub-layer quark model, in which the proton and the neutron are made up of an infinite number of point-like (structure-less) quarks  $u_{\infty}$  and anti-quarks  $u_{\infty}^{CP}$  at an infinite sub-layer level. In this paper, we propose that the dark matter is also made of an infinite number of quarks  $u_{\infty}$  and anti-quarks  $u_{\infty}^{CP}$ . A pair of the ultimate quarks  $u_{\infty}$  and anti-quarks  $u_{\infty}^{CP}$  would be produced in the first moments after the Big Bang and then remain as the dark matter for all time, stable against decay and subject only to the weak interaction and gravity. It is then shown that *CP* is violated in the doublet of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks to account for the asymmetry of the number of particles and anti-particles in the present universe. Furthermore, it is shown that the Higgs bosons are composed of  $u_{\infty}$  and  $u_{\infty}^{CP}$  dark matter particles and give the masses to gauge bosons, quarks and leptons.

Keywords Dark matter · CP violation · Higgs bosons

# 1 Introduction

All ordinary matter in the universe is composed of baryons, such as nucleons, atoms, molecules, gas clouds, planets, stars and galaxies. The most recent WMAP (Wilkinson Microwave Anisotropy Probe) observations strongly imply that ordinary matter comprises only

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4% of the universe and the remaining universe is made up of 22% dark matter and 74% dark energy [17]. If most dark matter is not baryonic, what is the dark matter made of? Likely relics of the early universe are species of stable, weakly interacting particles. As one model, we propose an exotic and non-baryonic quark with all one-half quantum numbers.

In some previous papers [7, 8, 14, 15, 18, 19, 25, 26], we proposed the infinite sublayer quark model in the framework of the standard  $SU(2)_L \times U(1)$  electroweak model.

This model implied that the proton (p) and the neutron (n) are made of  $u_1$  and  $d_1$  quarks, so that  $p = u_1u_1d_1$  and  $n = u_1d_1d_1$ . Furthermore,  $u_1$  and  $d_1$  quarks are made of  $u_2$  and  $d_2$ , etc. In summary,  $u_N$  and  $d_N$  quarks at level N are made of  $u_{N+1}$  and  $d_{N+1}$  quarks at level N + 1, such as  $u_N = (u_{N+1}, u_{N+1}, d_{N+1})$  and  $d_N = (u_{N+1}, d_{N+1}, d_{N+1})$  where N = $1, 2, 3, ..., \infty$ .

Here, the  $u_N$  and  $d_N$  quarks have quantum numbers of spin S = 1/2, isospin I = 1/2, third component of isospin  $I_3 = \pm 1/2$ , fractional electric charge  $Q = \frac{1\pm 3^N}{2\times 3^N}|e|$  (*e* is the electron charge), and baryon number  $B = 1/3^N$ . The antiparticle of  $u_\infty$  is the  $d_\infty$  quark, since the baryon number vanishes at  $N = \infty$ . The number of quarks at level N is  $3^N$ . Thus at  $N = \infty$ , an infinite number of point-like quarks ( $u_\infty$ ) and anti-quarks ( $d_\infty = \overline{u_\infty}$ ) is considered as constituting the nucleon. The ultimate particle  $u_\infty$  has quantum numbers of S = 1/2, I = 1/2,  $I_3 = 1/2$  and Q = (1/2)|e|. Thus, all quantum numbers of the  $u_\infty$  quark are just one-half and this quark is non-baryonic, since the baryon number B is zero at an infinite sub-layer level.

In the following, we shall summarize dark matter candidates which were proposed by many authors and propose the non-baryonic and exotic quark  $u_{\infty}$  as an excellent candidate for dark matter.

## 2 Dark Matter Candidates

Dark matter candidates are usually divided into two broad categories: baryonic dark matter and non-baryonic dark matter. The composition of baryonic dark matter is considered to consist of astronomical bodies such as black holes, neutron stars, white dwarfs, very faint stars, or non-luminous objects like planets (collectively called MACHOs or Massive Compact Halo Objects [1]) and cloud of non-luminous gas. Recent measurements of the cosmic microwave background radiation, as well as arguments based on Big Bang nucleonsynthesis and the growth of structure in the universe favor models in which the primary component of dark matter is new elementary particles outside the standard model of particle physics, collectively called non-baryonic dark matter. If most of the universe is in baryons, we encounter serious difficulty in explaining the observed structure formation [16]. The non-baryonic dark matter is divided into hot dark matter and cold dark matter, depending on their respective masses and speeds. Hot dark matter candidates move rapidly, while cold dark matter candidates travel at slow speeds or have little pressure.

Although there are a number of ideas as to what the non-baryonic dark matter consists of, we don't have any confidence at all as to what it consists of [3, 4]. The leading possibilities are:

- Neutrinos—neutrinos are now known to have very small mass and one of hot dark matter candidates [5].
- \* Axions—axions have been conjectured as an explanation for the slight breaking of *CP* asymmetry in particle physics [12]. Although axions could be extremely light, they could be numerous enough to contribute meaningfully to nonbaryonic cold dark matter [13].

Neutralino—neutralino is predicted to exist on the basis of supersymmetry, as supersymmetric counterpart of neutrino.

In many models, as a heavy, stable, the lightest supersymmetric particle (LSP) is an excellent candidate to comprise the universe's cold dark matter. A lightest neutralino of roughly 10–10000 GeV is the leading weakly interacting massive particle (WIMP) dark matter candidate [6]. The conservation of *R*-parity means that the LSP is absolutely stable [2]. This could turn out to be a good candidate for cold dark matter. *R*-parity is a multiplicative quantum number where all the particle of the standard model have positive *R*-parity, while their superpartoners in supersymmetry have negative *R*-parity, where the quantum number is given by

$$R = (-1)^{3B+L+2S} \tag{1}$$

for a particle with spin S and baryon and lepton number B and L.

Since *R*-parity is a multiplicatively conserved quantum number, the particle can only appear quadratically in the Lagrangian. This means that the supersymmetric particles can only be produced in pairs. Since this particle will be absolutely stable, there is no lighter charged particle into which it can decay. This fact is what makes the supersymmetric particles dark matter candidates.

Here we consider the ultimate quarks  $u_{\infty}$  and antiquarks  $u_{\infty}^{CP}$  as cold dark matter candidates to comprise the universe, since they are absolutely stable and the non-baryonic particles with the baryon number 0. In this model, a pair of an infinite number of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks can be produced thermally in the hot early universe of the Big Bang and leave approximately the right relic abundance to account for the observed dark matter.

First, we shall show that *CP* is violated in only one doublet of the ultimate quarks  $u_{\infty}$  and anti-quarks  $u_{\infty}^{CP}$  to account for the asymmetry of the number of particles and antiparticles in the present universe.

Next, we shall show that the present universe is full of the ultimate quarks  $u_{\infty}^{CP}$  and antiquarks  $u_{\infty}^{CP}$ . The gauge and Higgs bosons are composed of the ultimate quarks  $u_{\infty}$  and anti-quarks  $u_{\infty}^{CP}$ . Thus, the composite Higgs bosons can give the masses to gauge bosons, quarks and leptons in the framework of the standard  $SU(2)_L \times U(1)$  electroweak model.

#### 3 CP Violation in Dark Matter Candidate Particles

In the standard model, the left-handed weak isospin doublet of quarks and right-handed singlet are written as

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, u_R, d_R. \tag{2}$$

The symmetry between the left-handed and right-handed quarks is broken. The Lagrangian describing the electroweak interactions of u and d quarks is written as follows:

$$\begin{split} L &= -\frac{1}{4} \mathbf{W}_{\mu\nu} \cdot \mathbf{W}^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\chi}_L \gamma^{\mu} \bigg( i \partial_{\mu} - \frac{g}{2} \tau \cdot \mathbf{W}_{\mu} - g' \frac{Y}{2} B_{\mu} \bigg) \chi_L \\ &+ \bar{u}_R \gamma^{\mu} \bigg( i \partial_{\mu} - g' \frac{Y}{2} B_{\mu} \bigg) u_R + \bar{d}_R \gamma_{\mu} \bigg( i \partial_{\mu} - g' \frac{Y}{2} B_{\mu} \bigg) d_R \\ &+ \bigg| \bigg( i \partial_{\mu} - \frac{g}{2} \tau \cdot \mathbf{W}_{\mu} - g' \frac{Y}{2} B_{\mu} \bigg) \phi \bigg|^2 - V(\phi) \end{split}$$

$$-\left(g_d\tilde{\chi}_L\phi d_R + g_u\bar{\chi}_L\bar{\phi}u_R + H.c.\right),\tag{3}$$

where g and g' are coupling constants of SU(2) and U(1), respectively,  $\mathbf{W}_{\mu}$  are three gauge fields of SU(2),  $B_{\mu}$  is the gauge field of U(1),  $\frac{\tau}{2}$  are the generators of SU(2) and Y is the weak hypercharge. Furthermore,  $g_d$  and  $g_u$  are coupling constants of d and u quarks with Higgs boson.  $\phi$  is the Higgs doublet and  $\tilde{\phi}$  is its complex representation.

In a previous paper [27] and in our book [28], to explain *CP* violation in only one family of *u* and *d* quarks, we considered non-commutative SU(2)-bundle internal structure. To this end, we introduced a phase factor  $e^{i\delta}$  into the left-handed weak isospin doublet, viz.,

$$\begin{pmatrix} u_L \\ e^{i\delta}d_L \end{pmatrix}, u_R, d_R.$$
(4)

Now we consider *CP* violation in the doublet of the ultimate quarks  $u_{\infty}$  and  $u_{\infty}^{CP}$ . Then we replace *u* and *d* in (2) and (3) by

$$u \Rightarrow u_{\infty}, \qquad d \Rightarrow u_{\infty}^{cp}.$$
 (5)

Then (2) is written as

$$\begin{pmatrix} u_{\infty L} \\ u_{\infty L}^{cp} \end{pmatrix}, u_{\infty R}, u_{\infty R}^{cp}.$$
(6)

In some previous papers [7, 8, 15, 18–20], we applied the electroweak model of  $SU(2)_L \times U(1)$  to the interactions at the infinite sub-layer quark model. To account for *CP* violation, we shall consider the quantum numbers of weak isospin  $t_3 = 1/2$  and  $t_3 = -1/2$  for the ultimate particles  $u_{\infty L}$  and  $u_{\infty L}^{CP}$ , where  $u_{\infty L}^{CP}$  means the left-handed particle operated upon by charge conjugation *C* and then by parity *P*, viz.,

$$u_{\infty L}^{CP} \equiv \gamma^{0} C \gamma^{0} \frac{1}{2} (1 - \gamma_{5}) u_{\infty}^{*}.$$
(7)

At the infinite sub-layer quark model, the hypercharge of  $u_{\infty L}$  and  $u_{\infty L}^{CP}$  quarks becomes zero by applying the Nishijima-Gell-Mann relation to weak quantum numbers.

Now we consider the doublet  $\chi_L = (u_{\infty L}Au_{\infty L}^{CP})^T$  where the superscript *T* means transposed. By considering the internal structure which is described by the *SU*(2) noncommutative geometry, even if the *A* is a scalar then we can show that *CP* is violated.

As an example, suppose that  $A = e^{i\delta}$  where the phase  $\delta$  is an ordinary number, then  $\chi_L = (u_{\infty L} A u_{\infty L}^{CP})^T$  is written as

$$\chi_L = \begin{pmatrix} u_{\infty L} \\ e^{i\delta} u_{\infty L}^{cp} \end{pmatrix}.$$
 (8)

It is important to note that the phase factor  $e^{i\delta}$  cannot be eliminated by redefining the phase if we consider the internal structure which is described by the SU(2) noncommutative geometry. That is, the internal space operated by SU(2) is called the representation space mathematically. This is a two-dimensional vector space over the complex Körper C. Here we deform this space and assume a plane with the periodical boundary in each coordinate direction. After all, the internal space is described by the deformed SU(2)-bundle, in which the fiber associate in time-space is torus. Thus we consider the periodical condition on a twodimensional vector space. By imposing the non-commutative conditions on two periodical functions on torus, we consider *CP* violation from the non-commutative internal structure on such space.

Let the spinor components of quarks  $u_{\infty}$  and  $u_{\infty}^{CP}$  be  $u_{\infty\alpha}$  and  $u_{\infty\beta}^{cp}$ . These are the coordinates in the non-commutative internal structure. Thus we obtain

$$u_{\alpha\alpha}^* u_{\alpha\beta}^{cp} + u_{\alpha\beta}^{cp} u_{\alpha\alpha}^* = k_{\alpha\beta}, \qquad (9)$$

where  $u^*$  is the complex conjugate of u and  $k_{\alpha\beta}$  is a non-zero c-number. The plus sign came from the fact that  $u_{\infty\alpha}$  and  $u_{\infty\beta}^{cp}$  are Grassmann numbers. If we change the phase of the first isospin component  $u_{\infty\alpha}$  in (8) and redefine  $u'_{\infty\alpha} \equiv e^{i\delta}u_{\infty\alpha}$ , the constant  $k_{\alpha\beta}$  in (9) should change. This is not allowed. Therefore, *CP* is violated in the doublet of  $u_{\infty L}$  and  $u_{\infty L}^{cp}$ quarks in (8).

Next, we consider the case that the internal structure is described by the ordinary commutative SU(2) geometry, but A is a matrix formed by  $\gamma$  matrices. To satisfy the  $SU(2)_L$ gauge symmetry, the matrix A must satisfy the following condition [23]:

$$ACA^*C = -I_4,\tag{10}$$

where  $I_4$  is a 4 × 4 unit matrix [9]. For example, the matrix A is written follows [11, 23]:

$$A = \gamma_5 \gamma^0 \exp(id_\mu \gamma^\mu), \tag{11}$$

where  $d_{\mu}$  is a real vector in Minkowski space and independent of space.

Thus, we can show *CP* violation in the doublet  $\chi_L = (u_{\infty L}Au_{\infty L}^{CP})^T$  at an infinite sublayer quark model. We applied (11) to *CP* violation in  $\beta$ -decay by considering the preon  $a_L$ and  $a_L^{CP}$  where  $a_L = u_{\infty L}$  and  $a_L^{CP} = u_{\infty L}^{CP}$  [11, 24]. We have also considered the preon model in which preon  $(a_L)$  and antipreon  $a_L^{CP}$  construct the *SU*(2) weak-isospin doublet [9–11, 20–24]. Especially, it was then shown that quarks and leptons are made of three preons and gauge bosons of two preons [20, 26], there are just four families [22], there is a possibility of eliminating the preon self-energy divergences [9] and anomalies are eliminated in *CP* violation preon model [15].

#### 4 Higgs Bosons and Dark Matter Candidate Particles

The present universe is full of dark matter candidate particles  $u_{\infty}$  and  $u_{\infty}^{CP}$ . Here we will show that the Higgs bosons are composed of dark matter candidate particles  $u_{\infty L}$  and  $u_{\infty L}^{CP}$  quarks.

In a previous paper [8], we considered that gauge bosons are composed of the ultimate particles  $u_{\infty L}$  and  $u_{\infty L}^{CP}$  quarks at an infinite sub-layer quark level as follows:

$$\mathbf{W}^{+} = (u_{\infty L} u_{\infty L}), \qquad \mathbf{W}^{-} = (u_{\infty L}^{CP} u_{\infty L}^{CP}), \qquad \mathbf{Z}^{0} = (u_{\infty L} u_{\infty L}^{CP}).$$
(12)

In the following, we shall consider the composite model of the Higgs bosons. Consider the weak isospin doublet  $\chi_L = (u_{\infty L} u_{\infty L}^{CP})^T$  in the  $SU(2)_L \times U(1)$  gauge transformation. Then, the Lagrangian is invariant under the following infinitesimal gauge transformation:

$$\chi' = \left(1 + i\frac{g}{2}\alpha \cdot \tau\right)\chi,\tag{13}$$

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where g is the coupling constant,  $\alpha$  are parameters in  $SU(2)_L$  and  $\tau/2$  are generators of  $SU(2)_L$ . It was shown that the weak isospin doublet  $(u_{\infty L}u_{\infty L}^{CP})^T$  does not give any limitations to the parameters  $\alpha$ . Here the quark  $u_{\infty L}$  has quantum numbers of weak isospin t = 1/2, third component of weak isospin  $t_3 = 1/2$ , hypercharge Y = 0 and electric charge  $Q = \frac{1}{2}|e|$  while the  $u_{\infty L}^{CP}$  quark has quantum numbers of t = 1/2,  $t_3 = -1/2$ , Y = 0 and  $Q = -\frac{1}{2}|e|$ .

Now consider the right-handed quark  $u_{\infty R}$  with quantum numbers of  $t = 0, t_3 = 0, Y = 1$ and  $Q = \frac{1}{2}|e|$ . Under the infinitesimal gauge transformation, we obtain

$$u'_{\infty R} = \left(1 + i\frac{g'}{2}\beta\right)u_{\infty R},\tag{14}$$

$$\overline{u'_{\infty R}} = \overline{u_{\infty R}} \left( 1 - i\frac{g'}{2}\beta \right), \tag{15}$$

where g' is the coupling constant of U(1) and  $\beta$  is the parameter in U(1). From  $u_{\infty L}$  and  $u_{\infty R}$  quarks, we construct the Higgs scalar  $\phi$  of  $SU(2)_L \times U(1)$  symmetry as follows:

$$\phi = \begin{pmatrix} \overline{u_{\infty R}} & u_{\infty L} \\ \overline{u_{\infty R}} & u_{\infty L}^{CP} \end{pmatrix},\tag{16}$$

where the electric charge of  $(\overline{u_{\infty R}} u_{\infty L})$  is 0 and the electric charge of  $(\overline{u_{\infty R}} u_{\infty L}^{CP})$  is -|e|. Under the gauge transformation of  $\chi$  and  $u_{\infty R}$  in (13) and (14), the Higgs scalar  $\phi$  transforms as

$$\begin{pmatrix} \overline{u_{\infty R}} & u_{\infty L} \\ \overline{u_{\infty R}} & u_{\infty L}^{CP} \end{pmatrix}' = \left( 1 + i \frac{g}{2} \alpha \cdot \tau - i \frac{g'}{2} \beta \right) \begin{pmatrix} \overline{u_{\infty R}} & u_{\infty L} \\ \overline{u_{\infty R}} & u_{\infty L}^{CP} \end{pmatrix}.$$
 (17)

This corresponds to the  $SU(2)_L \times U(1)$  gauge transformation of the field with t = 1/2 and Y = -1. Therefore, the Lagrangian describing the isospin doublet, which is constructed from  $u_{\infty L}$  and  $u_{\infty R}$  quarks, is invariant under the  $SU(2)_L \times U(1)$  gauge transformation.

In the following, we shall consider the following Higgs potential  $V(\phi)$  to give the mass to gauge bosons and leptons:

$$V(\phi) = \mu^2 \phi^{\dagger} \phi + \lambda \left( \phi^{\dagger} \phi \right)^2, \qquad (18)$$

where  $\mu^2$  is the mass parameter and  $\lambda$  is the coupling constant.  $\mu^2 < 0$  and  $\lambda > 0$ . Then,  $V(\phi)$  becomes a minimum when

$$\phi^{\dagger}\phi = \left|\overline{u_{\infty R}}\,u_{\infty L}\right|^2 + \left|\overline{u_{\infty R}}\,u_{\infty L}^{CP}\right|^2 = -\frac{\mu^2}{2\lambda}.$$
(19)

Putting

$$v^{2} = -\frac{\mu^{2}}{\lambda} = 2\left(\left|\overline{u_{\infty R}} \, u_{\infty L}\right|^{2} + \left|\overline{u_{\infty R}} \, u_{\infty L}^{CP}\right|^{2}\right) \tag{20}$$

and defining the vacuum expectation value  $\phi_0$  of  $\phi$  as

$$\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}. \tag{21}$$

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The Higgs potential produces the spontaneous symmetry breakdown as well as the usual standard model. In this case, quantum numbers of  $\phi_0$  have t = 1/2,  $t_3 = 1/2$  and Y = -1, so that the electric charge Q = 0.

Define the Higgs field  $\phi$  by

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v+h\\0 \end{pmatrix},\tag{22}$$

where *h* is the piece over and above the vacuum. Thus, the vacuum expectation value in (21) and (22) gives the masses to gauge bosons, quarks and leptons. We have already considered the preons  $a_L$  and  $a_L^{CP}$  where  $a_L = u_{\infty L}$  and  $a_L^{CP} = u_{\infty L}^{CP}$ , and obtained the same results [21].

#### 5 Conclusions

We proposed the infinite sublayer quark model, in which there exists an infinite number of quarks  $(u_{\infty})$  and anti-quarks  $(u_{\infty}^{CP})$  at an infinite sub-layer level. The ultimate quarks  $u_{\infty}$ and  $u_{\infty}^{CP}$  are the exotic, structure-less, stable and non-baryonic particles with all half quantum numbers. An infinite number of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks was created in the early universe after the Big Bang. Then we can show *CP* violation in the doublet of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks to account for the asymmetry of the number of particles and anti-particles in the present universe. The abundant relics of an infinite number of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks are full of the present universe and form the dark matter. The Higgs bosons are composed of the dark matter particles of  $u_{\infty}$  and  $u_{\infty}^{CP}$  quarks, and give the masses to gauge bosons, quarks and leptons.

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